Deepening Students' Fractions Problem Solving Skills

A Prototype of Deeper Learning Adaptive Math Tools and Culturally Relevant Stories

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Table of Contents

EXECUTIVE SUMMARY 3

Key Findings 3

RATIONALE 3

Supporting Deep Math Learning 4

Fractions Understanding is Key 4

Focus on Equity 5

A Role for Technology 5

CO-DESIGN PROCESS 5

THE PROTOTYPE 7

RESEARCH 8

Research Questions 8

Methods 8

Participants 9

Measures 9

PROMISING RESULTS 10

Design-based results 10

Initial evidence of impact 12

Positive teacher response 15

DISCUSSION AND FUTURE WORK 16

EXECUTIVE SUMMARY

With generous funding from the Bill & Melinda Gates Foundation, Teachley has developed a digital problem-solving environment, where 3rd-5th grade students create and manipulate representations of fractions as they solve problems. Teachley collaborated with a community-based organization and teachers who work with low-income students of color to co-design the problem content and research the impact of the software on students' fractions problem-solving skills.

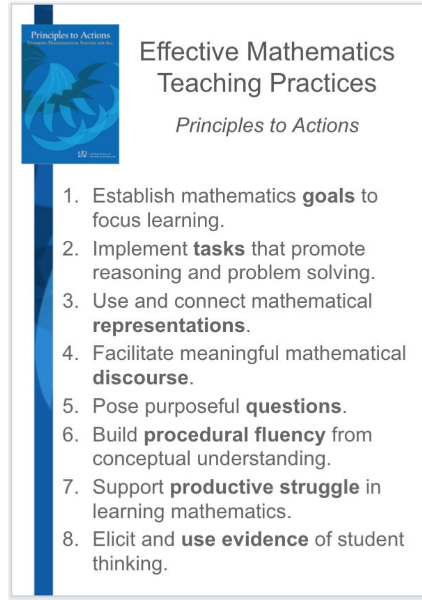
Key Findings

- 140 students in grades 3-5 used the fractions problem-solving software during their regular math class for 3-5 sessions. Students' fractions problem-solving skills **significantly improved by 20 percent** after using the software.
- Teachers believe the software is **easy to implement** in a classroom setting, **aligns with their school's goals**, and **has the potential to improve student outcomes**.
- Teachers, children, and their families believe the math stories are **culturally relevant and reflective of their own experiences**.

RATIONALE

Mathematical problem solving is at the heart of effective mathematics learning because it requires students to think deeply about content and employ higher-order thinking (Schoenfeld, 1985). Because of this, problem solving is an extremely complex topic for students and teachers (Carpenter, Fennema, Peterson & Carey, 1988; Tambychik & Meerah, 2010). For students, solving problems requires that they make sense of the problem, distinguish between useful and irrelevant knowledge, choose from a range of strategies, and calculate the answer (Jitendra, et al. 2013; Schoenfeld, 1985). Teachers need to support students who may fail to grasp the content, struggle with higher-order thinking and have difficulty explaining their thinking. Elementary teachers are typically underprepared to teach deep mathematical content (Ball, 2015) and often experience math anxiety (Hembree, 1990), which make it exceedingly challenging for them to support students and encourage strong math conversations.

Supporting Deep Math Learning



The National Council of Teachers of Mathematics (NCTM) has identified eight *Effective Mathematics Teaching Practices* in which they highlight the importance of engaging students in problem solving tasks that promote deep thinking and facilitate meaningful discussions (Leinwand, 2014). These tasks should allow students to share their reasoning, use multiple strategies, and build collective understanding through analysis of their peers' work. Teachers should ask questions that allow them to "assess and advance students' reasoning and sense making about important mathematical ideas and relationships" and use evidence of student thinking to assess understanding and adjust instruction accordingly (Leinwand, 2014, p.3).

Fractions Understanding is Key

An equally pressing issue facing U.S. schools is the enormous deficit in students' understanding of fractions. The National Math Advisory Panel (2008) found that proficiency with fractions is the "most important foundational skill not presently developed [by students]" (p. 18). Half of U.S. eighth graders are not able to correctly order three fractions (IES, 2011), a fourth grade Common Core standard. Understanding fractions is challenging because rational numbers seem to operate differently than whole numbers (Siegler, Thompson, & Schneider, 2011). For example, with whole numbers, multiplication increases the resulting quantity while division decreases it, yet with fractional numbers smaller than one, the opposite is true.

Fraction knowledge is strongly related to future academic performance; thus, early intervention is crucial. Research shows that students who start middle school with poor understanding of fractions are more likely to struggle with later mathematics. For instance, Siegler et al. (2012) found that fifth grade fraction knowledge predicts high school math performance, even when controlling for working memory, whole number knowledge, IQ, reading ability, and demographic factors. However, low-performers can improve their understanding of fractions and narrow the achievement gap (Fuchs et al., 2013).

Since the core of fractions instruction occurs in grades 3-5, it is elementary school teachers who are primarily responsible for teaching this essential content. However, many U.S. elementary school teachers lack sufficient fractions content knowledge (Newton, 2008). Additionally, current mathematics curricula focus on teaching rational numbers as a unique system separate from whole numbers; often failing to highlight the relationship between fractional and whole numbers (Fuchs et al., 2013). Using effective visual models like the number line is crucial for improving students' understanding of this challenging content, yet teachers are not equipped with effective instructional materials.

Focus on Equity

Black, Latinx, and low-income students often face additional challenges (e.g., stigma, low-rigor curriculum, teachers' low expectations) that elevate the importance of rich problem solving experiences that are challenging and promote a sense of value and belonging in mathematics (Matthews, 2018). Culturally relevant pedagogy should incorporate real-world problems that align with students' lives and allow students opportunities to solve problems using different strategies (Tate, 1995). NCTM underscores the need for equity-based instruction which involves *reflecting* on one's work and identity, *noticing* different ways students express their thinking, and *engaging in community* to bridge the gap between students' lives and the mathematical learning experiences within the classroom (Chao, Murray, & Gutiérrez, 2014).

A Role for Technology

Technology offers an exciting avenue for improving students' fraction problem solving, as it has the potential to provide interactive visual models and feedback, which can clarify concepts, enable supported practice, and encourage metacognition (awareness of one's own thinking). Additionally, software can make it easier for teachers to understand children's mathematical thinking, help students explain their solutions, and engage students in rich conversation about deep mathematical ideas. While these goals are ambitious and challenging, Teachley has extensive experience developing software centered on promoting conceptual understanding and the use of research-based strategies. Our existing math interventions, focused on single-digit operations and, more recently, fractions have significantly enhanced student outcomes (Carpenter, 2017; Pagar, 2017; Carpenter, 2018) and are widely adopted by schools and districts across all 50 states and internationally.

CO-DESIGN PROCESS

In order to engage teachers and students directly in the design and development process, Teachley partnered with two charter schools in East Harlem and Bronx, NY that serve primarily

Black, Latinx and low-income students. Working with these schools helped us to include different perspectives and lived experiences into our designs and to explore the supports needed for typical school implementations. Focus groups with children and parents helped us assess whether the math stories we created made sense and were relevant to their lives. Teacher interviews provided insight about typical practice and classroom needs, and how well our problem content aligned with those. We also collaborated with the Hispanic Information and Telecommunications Network (HITN), the nation's first Latino-controlled, commercial, Spanish-language media company. HITN co-wrote the problem content with Teachley, led the parent and student focus groups, and translated all content into Spanish.

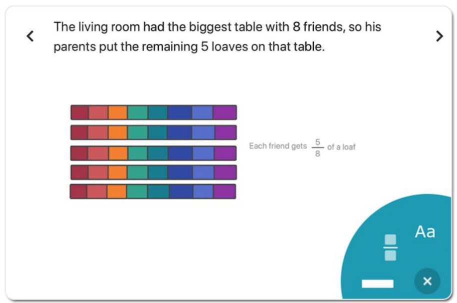
Coherent Design. Developing high-quality math software that works in real classrooms is complex and requires designers to understand cognitive development, mathematics pedagogy, elementary classrooms, children's diverse backgrounds, and technological affordances. Teachley's work is strongly guided by research from the learning and cognitive sciences and best practices in math education. We design for diverse learners, especially for those living in communities that have typically been underserved, as well as for those who struggle in mathematics or may be at-risk for learning disabilities.

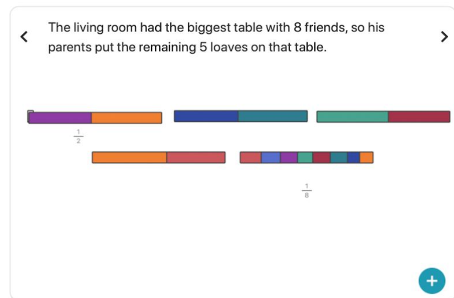
Rapid Development. Throughout our design process, we work with children to field test designs. This process allows the development team to see intended users interact with the software, giving them better insight into useful features and needed modifications. Using agile development, we conduct iterative component testing and focus on creating high priority development goals. For example, we might build a prototype with one key feature, test the component in classrooms and review it as a team before modifying it or developing additional features. Field testing early and regularly allows us to make design decisions efficiently and incorporate feedback from classrooms.

Multi-faceted research. We incorporate research in different ways, such as initial usability testing, focus groups, interviews, pre/posttest assessments, and randomized control studies. Conducting observations of classes using the software is a powerful method for evaluating initial effectiveness of designs. At an early stage, the research is highly formative and focuses on assessing the usability of the software and children's understanding of the content and tools. Later evaluations measure the impact made on learning by looking at change in performance before and after using the software.

THE PROTOTYPE

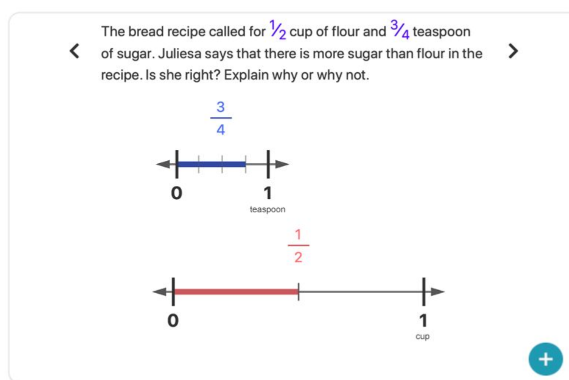
The broad aim of our project is to transform classroom math practice, so that in all classrooms children are engaged in complex problem solving with rich math conversations that deepen understanding and support peer-to-peer learning. We expect that giving students a digital problem-solving environment with culturally relevant math stories will 1) help students see themselves connected to math, 2) support them in creating stronger math models and 2) collect data that can tap into their thinking, strategies, and process. The software can then enhance teachers' ability to assess their students' thinking and plan more effective discussions.





During this initial phase of work, we created a prototype of student-facing tools focused on fractions representations. The prototype includes problems and tools for four fractions content areas: 1) comparison, 2) addition/subtraction, 3) multiplication, and 4) extended scenarios. Across the content areas, students can drag words and numbers directly from the problem text into the workspace. They can add number lines, bars, and area models, which can be connected to fractions dragged from the problem text or new fractions the student creates.

A common misconception is that fractions have an absolute size, when in fact, like whole numbers, fractions are relative to the unit size, with ½ a mile being much longer than ¾ of a yard. Within the software, students can identify the unit associated with a given number, for example, by dragging the word "yard" from the problem to answer the question "¾ of what?" The resulting model for that number reflects the unit, for example, the number line becomes labeled 1 yard, 2 yards, etc. When comparing fractions with different units (e.g. ¾ tsp and ½ cup), students can adjust the relative size of their models to show that even though ¾ > ½, when you consider the unit, ¾ of a tsp is much smaller than ½ of a cup.



The software tools have additional features for adding, subtracting or multiplying fractions. For example, a student can use an equivalence tool to show how 1/2 is the same as 2/4, which can then be combined with 3/4. Students can use area models to explore how multiplying by a fraction less than 1 results in a smaller number, while multiplying by a fraction greater than

1 results in a bigger number. We also created extended scenarios, which involve more open-ended problem solving, where students may take several class periods to generate a solution.

RESEARCH

Initial research goals of the prototype included design-focused research questions and questions related to the impact on student learning.

Research Questions

Design-based.

What supports/tutorials do students need to understand how to use the toolset?

Do students understand the questions being asked? Do students and their families relate to the content?

Impact on student learning.

Do students demonstrate a higher ability to solve fractions comparison problems after using the toolset? How do these results differ based on students' demographics?

How easily are students able to use the toolset to model their problem solving process, regardless of whether their model or answer is accurate or inaccurate?

Methods

Initial testing and focus groups. To evaluate early builds of the software, we worked one-on-one with 24 students in grades 3–5 approximately once a week during an after school program. To explore the cultural relevance and how well the problems reflect experiences of families and communities, HITN facilitated focus groups with students and parents/grandparents.

Pilot studies. We also conducted two research studies, Study 1, which examined the effects of the comparison toolset, and Study 2, which examined how well students could use the toolset to solve an extended scenario. In Study 1, six classrooms in grades 3-5 used the software during typical math instruction (~50 minutes) once per week for five weeks in May-June 2019 after the teachers had finished their fractions units. Third and fourth graders used the comparison toolset for all five sessions while fifth graders worked on comparison problems for three sessions, took the posttest, then used the fractions add/subtract toolset for the remaining two sessions. In Fall 2019, we conducted Study 2, where four 4th and 5th grade classrooms used the software over two class periods to model various solutions to a multi-step problem.

At the start of each session, facilitators taught a mini-lesson which served as an introduction to the software and math content, circulated the room during the independent/partner work, and reconvened the class for a wrap-up discussion. Each facilitator had a deep understanding of the prototype and held prior experience teaching. Students worked on three to five problems each session, reviewed their work with a partner, and recorded answers in a paper-based workbook. The goal of the workbook was to explore scaffolds that might be useful to embed in the software. Also, early versions of the prototype did not save students' work, so the workbook provided students with an enduring visual representation of their models. During the discussions, 1-2 students demonstrated how they solved a problem by connecting their computer to the Smartboard and talked through their process. A trained observer from the research team recorded field notes and helped with difficulties or questions. Classroom teachers also participated in the sessions; some teachers were more hands on and involved in helping their students complete the problems while others primarily observed.

Participants

During the research studies, we worked with two elementary charter schools predominantly serving Latinx, Black, and Afro-Latinx students from low-income households (99.9% qualified for free or reduced-price lunch) in the Bronx and East Harlem, New York. In the first study, there were 140 students in grades 3-5, while the second study included 63 4th and 5th grade participants (2 classes per grade level in each study).

Focus groups included ten 4th and 5th graders from the East Harlem school. The family focus group participants were recruited from a public K-5 school in Brooklyn, NY serving primarily low-income families of color (93% free/reduced lunch, 67% Black, and 19% Latinx). Eight mothers, two fathers, and two grandmothers participated in the family focus groups.

Measures

Study 1 included several measures:

- A paper/pencil pre-assessment of fractions problem solving skills, including multiple choice and extended response items from publicly released standardized measures.

- A posttest in which half of the items were completed digitally using the toolset and the other half using paper/pencil. Students were randomly assigned to complete one of two versions. All students began with the digital items and then completed the paper/pencil items.
- Piloting of a new measure of math enjoyment before and after using the software
- A survey about the Teachley software, administered during the final research session.
- Teacher survey, administered during the final research session.

Student work in Study 2 was evaluated using a rubric that evaluated students' effort, clarity, and mathematical accuracy of their models. Students completed a survey at the end of the focus groups. Full versions of the student measures are included in the Appendix.

PROMISING RESULTS

Design-based results

To evaluate the first two design-based research questions, researchers used a qualitative descriptive content analysis. This involved categorizing and summarizing data from on-going field testing of designs, focus groups, and classroom observations during the studies.

Do students understand the questions being asked? Do students and their families relate to the content?

In general, students **understood problems** being asked, **could model their thinking** independently, they found the questions **interesting and relatable.** Eight out of 10 students who participated in focus group discussions agreed problems were clear and relatable. They also raised interesting and valid points when asked during focus groups whether problems were confusing the focus groups. For example, while discussing a problem involving a teacher surveying students, one child asked *"Why is this woman teaching 3rd* ***and*** *4th grade?"* while another pointed out *"There's only like five people in one class!"* In the first study, the observers noted that some students found the content hard at times, an indication that the problems were appropriately challenging without being too difficult.

During the family focus group, parents and grandparents related to the math story content. For example, one story involved a child baking muffins with her grandmother. During the conversation, one mother explained how she purchased measuring cups specifically because her child liked to bake pancakes for their family. Families reported that sometimes it can be difficult to help children with math word problems at home, suggesting that problems should also consider adults' perspectives by connecting math problems to families' everyday experiences. At the completion of the focus groups, the parent coordinator commented, *"The* *parents had never experienced anything like this before, where their inputs felt important and mattered. And that what they had to say was going to make a difference."*

Although the primary goal of the teacher interviews was to explore the mathematical content of the math problems, the teachers repeatedly brought up how relatable the problems were. For example, one 4th grade teacher said: "***I like that you have a lot of things my kids can do, they're relatable. It definitely adds to (the stories)."*** while another commented, ***"because the problems are so relatable, it's easier for them to do it."*** Teachers appreciated that the names used in the problems were reflective of their students,*"Oh, I have a Jayleen in my class!"* She explained that the names often used in math problems are not reflective of her students. Teachers also noted how locations included in the problems (e.g., went to the deli) are reflective of real places their students go or things they could relate to. Adjustments to the content were made, as needed, based on focus group discussions with students, teachers, and parents.

What supports/tutorials do students need to understand how to use the toolset?

Initial and on-going field testing of the software helped the development team adjust the designs. For example, when creating their models, students initially had trouble distinguishing between placing/moving tick marks and coloring in segments to represent the fraction. To better highlight which mode the students were in, we built a preview feature, so that as students hover the mouse, they see a semi-transparent preview of what will happen when they click.

During the first study sessions, an observer coded *Most*, *Some*, *Few*, or *None* to describe the proportion of students who: 1) were on-task, 2) having difficulty using the toolset, 3) confused by the problem content or tools, and 4) able to work independently. In general, **students needed little support in getting started** with the toolset, and the **layout and features of the toolset were clear and user-friendly**. In 17 of the 25 observed sessions, few or none of the students had difficulties using the toolset. Five of the eight observations in which at least some students experienced difficulties were from a classroom with consistently disruptive behavior. Notably, a school administrator confirmed that the disruptions were typical for this particular class.

In both third grade classes, some children had difficulty during the first session. Instead of editing a model, they often deleted it and started over. This indicated an initial learning curve, especially for the youngest students. However, this finding was not surprising, given young students' relative inexperience using computers as well as noticeable challenges of Chromebook track pads. The observers noted that the few students who were confused tended to have difficulty with the math content, not the the software. For example, not all students were familiar with the strategy of comparing fractions to a landmark, so it was new for them to use that strategy with or without the technology. In general, the classroom observations provided useful ideas for types of scaffolds that could be embedded into the software to help students master more challenging content.

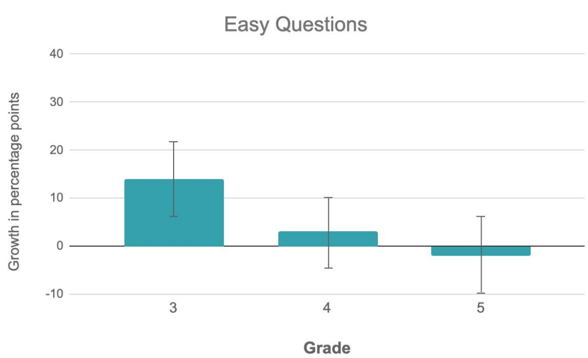
Facilitators structured follow-up mini-lessons to address misconceptions and deepen understanding in follow-on sessions. Students were able to gain familiarity with the software and tools quickly, with most students able to use the toolset independently in five of the six classrooms during the final two sessions.

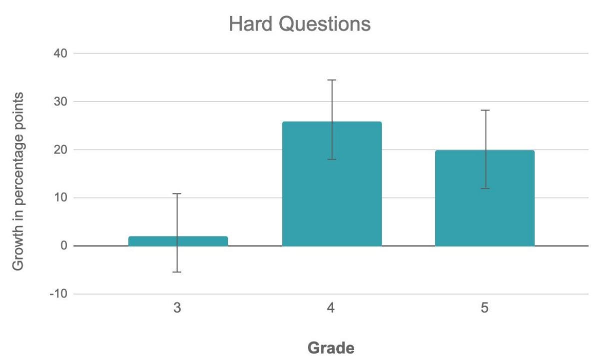
Initial evidence of impact

Do students demonstrate a higher ability to solve fractions comparison problems after using the toolset?

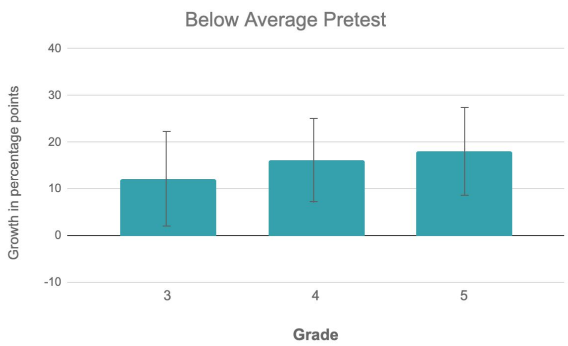
After the 3-5 sessions, students' posttest scores **significantly improved by 20%** (47% at pre-test to 57% at post-test, $p < .005$, effect size of 0.33). The comparison content was aligned with 4th grade, so we expected the largest improvement for that grade level. We expected less growth for 3rd grade since much of the content was above-grade level and 5th grade, because many had already mastered the content. The data confirmed this pattern: 4th grade improved 12 percentage points, 3rd grade improved 9 points, and 5th grade, 8 points. The 4th grade improved the most despite one of the two classes having disruptive behavior.

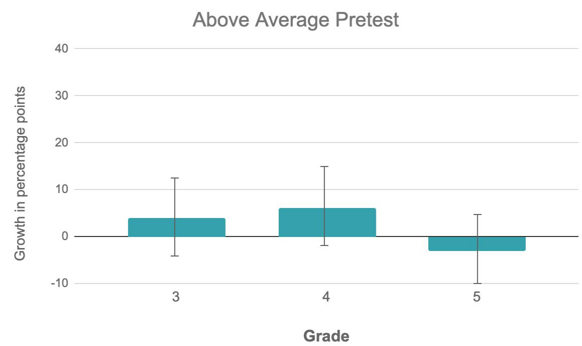
Student growth aligned with item content difficulty. Each item on the fractions assessment was coded either "easy" or "hard", based on grade level alignment of the content. This resulted in 4 "easy" items and 3 "hard" items. As anticipated, 3rd graders improved on easy questions, while 4th and 5th graders improved on hard questions.





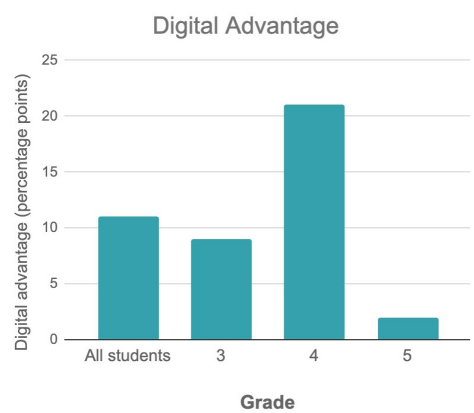
Growth driven by struggling learners. The data also suggests growth from pretest to posttest was largely seen by students who scored below the average on the pretest. Fourth and fifth graders who scored below the average are students who did poorly on the fractions comparison pretest despite 2-3 years of formal instruction on the topic. These were students who





benefited significantly from using the software. Students who scored above average on the pretest did not significantly improve their performance.

Digital Advantage. In order to examine whether using the software helped students during the posttest, we calculated a digital advantage by comparing the average growth from pretest to posttest for those who saw the item digitally versus on paper. The data suggests that all students experienced some digital advantage when completing items on the posttest using the toolset. Most prominently, average growth for 4th graders was 21 percentage points higher when questions were administered digitally than on paper.



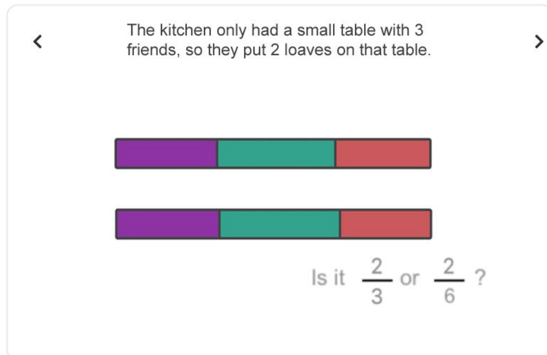
A more nuanced examination of digital advantage by item suggests that for some questions, the software helped make content they may not have yet learned in school more accessible. For example, third graders did 28% better on one hard question about equivalence when they created digital models. Yet, other assessment items were more challenging to complete using the toolset. For example, one item required students to model tenths and twelfths, which at the time of the first study, was time consuming and may have contributed to more errors. After the study, we created an "autoticker" feature that allows students to create models of fractions with large denominators more easily and accurately.

How do these results differ based on students' demographics?

There were no statistically significant differences found between girls and boys. Nearly all participants (99.9%) were low-income, so we were not able to disaggregate based on income. Also, nearly all participants were students of color (98% Black, Latinx, or Afro-Latinx).

How easily are students able to use the toolset to model their problem solving process, regardless of whether their model or answer is accurate or inaccurate?

In the second study, we examined how well students were able to model complex, multi-step fractions problems. We found that 95% of students were able to demonstrate proficiency or advanced application of their fractions skills to model their solutions. Importantly, students solved the same problem using many different models, indicating that the toolset can support a diverse range of approaches to applying fractions knowledge. The figure below shows two different student solutions for how to equally split 5 loaves of bread between 8 friends.



This is important because oftentimes, there is not only one unique correct method for solving extended response questions. Students' solutions paths can showcase a range in thinking, revealing misconceptions and errors along the way. For example, in one 5th grade class, a student successfully created models showing how to split 2 loaves of bread amongst 3 friends equally. However, she was not confident in how to write the corresponding fraction. Using the toolset, she was able to project her screen to the entire class at that moment and ask her peers whether she would label the fraction 2/3 or 2/6. The class discussed this problem for a few minutes, the student then felt confident in how to label her models (⅔), and everyone resumed their work. At the end of the research session, the classroom teacher gathered the class together to continue reviewing the concept brought up by the student. This example provides a clear demonstration of how the software can support teachers in gaining insight into students' thinking and help them identify areas for reteaching or further discussion.

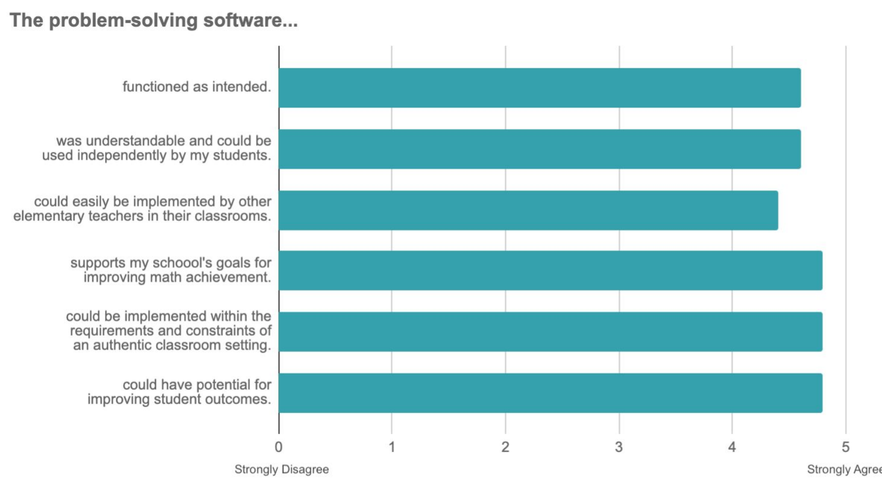
One teacher noted during a class share that the student projecting her work on screen had always refused to share before and that this was the first time she agreed to talk about her work. The student proudly stood at the front of the class and explained her model created using the toolset.

Math engagement. A close examination of the enjoyment measure suggested that the four questions belonged to a single construct, but that the internal reliability of the measure was not stable overtime. In particular, many of the students did not understand how to select just one response. There were too many incomplete or unusable reponses, especially at pretest, for this measure to be interpreted. For student surveys administered after the first study, we used a response format (adapted from the TIMSS, 2019) that was more reliable with this age group.

Student survey. Students in Study 1 positively agreed that the software: helped them learn math, allowed them to revise their work, was easy to use, and that, if given the choice, they would choose to use the Teachley software during math class.

Positive teacher response

Five teachers completed a survey at the end of the first study, and all agreed or strongly agreed that the fractions software is easy to implement in a classroom setting and believe it has the potential to improve student outcomes (see figure).



One teacher noted that the ability for students to represent a problem in multiple ways specifically supports their school's math goals. Another teacher explained that she liked that the students enjoyed using the software and that they remained engaged on problem solving with fractions. Most of the teachers wrote about how their students learned as a result of the software.

- *"The reactions from the students are very positive. Many students have said, "Wow, now it makes sense."*
- *"The things they are learning in the fraction tool set help the students with concepts that have been a struggle for them this year and last year."*
- *"My students tend to struggle with fractions, but with the assistance from the Teachley team, my students grasped a better understanding on how to solve problems involving fractions."*

Separately, we interviewed four different teachers about the story problem content and appropriateness across grade levels. In grade-level pairs, we met with two third and two fourth grade teachers for 45-60 minutes. As they read each story, they determined the grade level each problem would be appropriate for, whether the content was approachable for their students, if anything was confusing, and/or if they would change the math problems. The third grade team discussed a total of 14 problems while the fourth grade team discussed 28 problems.

Four key insights: In general, the teachers noted that the problems were well-written, clear, and appropriate for grades 3-5. They reported that the math problems would make a great addition to classrooms and indicated they would be interested in implementing the problems, with one 4th grade teacher responding, "*Definitely, I would teach these problems!"*

- **Distractors are good and even better when they add to the context of the story.** Word problems often present extraneous information, called distractors, which are irrelevant to solving the problem. Students have to be able to identify the specific information that *is needed* and ignore the rest. The teachers noted that the distractors in the Teachley problems added to the overall context of the problem, thereby making the problems more approachable and relevant for children to grasp.
- **Explanations are important.** The teachers noted how important it is to include an explanation prompt. They explained how these prompts mapped onto one of the problem solving steps they now use in their classrooms. They shared a story of how one of their students, who excels in math surprisingly scored low on the state math test. Upon review of the results, they found that while she was able to derive the correct answer, she could not then explain "why" and instead wrote a question mark as her answer.
- **Multi-step and lengthy problems are essential but can be challenging.** Teachers described that their students typically find multi-step problems difficult. For example, students may not attend to a secondary question or get confused by distrators, especially they said, for students learning English. Teachers recognized that despite the challenge, it is important that students be able to work through more complex problems. They recommended paying careful attention to the amount of text, the number of questions asked, and how questions are presented within the same story problem. They identified a few specific problems that could be adjusted.
- **Scenario problems would provide insight on students' thinking.** The intent behind the scenario stories is to elicit student collaboration and extended engagement in the same problem over a period of time. The teachers commented that the context of the scenarios would be approachable for their students (for example, buying birthday gifts that they actually want to buy in real life, deciding upon sports equipment for their school, etc.) thereby making them easier to get into. The teachers pointed out that it takes a lot of practice to get students to where they can engage in conversation about their work and that of their peers. They believed the scenario problems presented would allow them to engage more deeply with their students.

DISCUSSION AND FUTURE WORK

The results of this research highlight the powerful impact of Teachley's fractions problem-solving software on student learning and the potential for this software to positively affect learning, especially for low-income, Black and Latinx students. Students were able to model their thinking using novel digital tools, with minimal support.

"The students were engaged in solving the problems using the toolset. I believe when students are engaged they are able to retain more information."

- 3rd grade teacher

Students' problem solving scores at posttest were 20% higher than at pretest, suggesting strong growth in their understanding over the course of the intervention. Allowing students to use powerful visual models, like the number line and number bar, appeared to impact their approach to solving fractions comparison problems. They were able to build models of fractions, split them equally based on the denominators, and paint them to match the numerators. They were then able to compare the fractions to each other, by making sure the wholes were the same size, and answer the word problems successfully. Further studies should attempt to tease out which features and elements of the software impacted learning the most.

The first study focused on 4th grade fractions comparison content; therefore, we expected that the material would be challenging for 3rd graders and easier for 5th graders. The results showed that 4th graders made the most improvement over the course of the study. Importantly, 4th and 5th graders who did poorly on the pretest had already had 2+ years of formal fractions instruction, yet they scored worse than many third graders at pretest. These struggling learners made significant growth using the software, which is an early indication that the software may be particularly useful for narrowing the achievement gap.

In the second study, we sought to determine whether students were able to use the software to approach extended fractions scenarios and found that 95% of students were able to demonstrate proficiency or advanced application of their skills to model solutions. This finding suggests that with appropriate support and implementation, students were able to model their thinking using the software and accurately solve the problems. The diversity of students' models highlights the tools' affordances to support different ways of thinking and various strategies, fundamental goals of the original design. Since the facilitators directly supported students through mini lessons, feedback, and class discussions, future work should examine best practices for ensuring effective integration into classroom practice.

There are several limitations of this research to consider. Both studies used a one group design where students served as their own controls. We recognize that this type of design makes it hard to determine whether other factors, such as classroom instruction, may have impacted students' growth. However, the first study occurred at the end of the school year, after all fractions instruction had been completed in the participating school. Thus, we have a high degree of confidence that growth in students' fractions problem solving was primarily related to the use of the software, rather than outside instruction. Future studies should use a randomized control design to increase the internal validity of the research.

In both studies, trained facilitators led the interventions, introducing the content through mini-lessons and showcasing student work to guide discussions. Future research should replicate with a larger sample size of classrooms and include teachers as facilitators of the software during their typical classroom practices to better evaluate the feasibility of the intervention for schools and increase the generalizability of the results.

While the studies showed strong promising impact on student learning, expanding the development of the prototype into a complete intervention that includes a teacher dashboard will make it easier and more effective for teachers to integrate the software into their classroom practice. Teachers need quick, efficient ways to view and share their students' problem solving work in order to offer feedback, inform instruction, and guide class discussions. The teacher interviews validated the importance of these features. We plan to develop a teacher assessment dashboard that syncs with the software to help teachers assess student understanding and better prepare for class discussions and instruction.

Future iterations of the software will include professional development resources, such as videos of teachers using the software to lead class discussions. The resources could highlight common student misconceptions and offer suggested mini-lessons or extension activities that can be used alongside the software. Allowing students and teachers to create their own math problems is another exciting way to expand the content offerings and foster a stronger sense of belonging for students. We also intend to expand the content beyond fractions to whole number and decimal word problems.

For questions, please contact info@teachley.com

Appendix A - Study Measures

Fractions Problem Solving - Collection of released items relating to fractions comparison from the NAEP, MARS tasks, Engage NY, and STAAR.

Name ____ Date __________________

Directions - Circle the correct answer. Use the space on the right to draw your ideas.

Draw your ideas here.

1 An office had three baskets of letters ready to be mailed. The first basket was $\frac{2}{10}$ full, the second basket was $\frac{3}{6}$ full, and the third basket was $\frac{1}{5}$ full.

Which comparison is true?

A. $\frac{1}{5} > \frac{3}{6}$

B. $\frac{2}{10} = \frac{1}{5}$

C. $\frac{3}{6} < \frac{2}{10}$

D. $\frac{1}{5} > \frac{2}{10}$

2 Students in Mrs. Johnson's class were asked to tell why $\frac{4}{5}$ is greater than $\frac{2}{3}$.

Whose reason is best?

A. Kelly said, "Because 4 is greater than 2."

B. Keri said, "Because 5 is larger than 3."

C. Kim said, "Because $\frac{4}{5}$ is closer than $\frac{2}{3}$ to 1."

D. Kevin said, "Because 4 + 5 is more than 2 + 3."

Draw your ideas here.

3 Kim, Les, Mario, and Nina each had a string 10 feet long.

Kim cut hers into fifths.
Les cut his into fourths.
Mario cut his into sixths.
Nina cut hers into thirds.

After the cuts were made, who had the longest pieces of string?

A. Kim

B. Les

C. Mario

D. Nina

4 The table shows the height increases, in inches, of some girls in Gina's class from last month to this month.

HEIGHT INCREASES IN 1 MONTH

Name	Height Increase (inches)
Gina	$\frac{3}{8}$
Maxine	$\frac{2}{3}$
Shari	$\frac{2}{4}$
Vanessa	$\frac{3}{12}$

What girl had a height increase that was greater than $\frac{1}{2}$ inch?

A. Gina

B. Maxine

C. Shari

D. Vanessa

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all of your work.

5 José ate $\frac{1}{2}$ of a sub sandwich.

Ella ate $\frac{1}{2}$ of another sub sandwich.

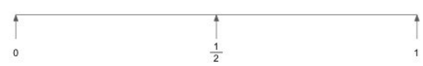
José said that he ate more sandwich than Ella, but Ella said they both ate the same amount. Use words and pictures to show that José could be right.

6 Mark says $\frac{1}{4}$ of his candy bar is smaller than $\frac{1}{5}$ of the same candy bar.

Is Mark right? Yes or No? _________________

Draw a picture or use words to explain why you think Mark is right or wrong.

7 Here is a number line.



Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? ________________________

Explain how you figured it out.

Math Enjoyment - Piloted during Study 1 (adapted from the West Virginia Mathematics Curriculum Impact Study and the Beliefs, Engagement, and Attitude Math Motivation Scale, Orosco 2016).

Name_____________ Date_________

On a scale of 1 to 5, where 1 is *Not at All True*, 3 is *Somewhat True*, and 5 is *Very True*, what best describes what you think of each statement A and B below.

	1 **Not at All True**	**2**	**3** **Somewhat True**	**4**	**5** **Very True**
A. I like math work that I'll learn from, even if I make a lot of mistakes.					
B. When I run into difficulty doing a math problem, I go back and work out where I went wrong.					

C. Which statement do you most agree with, 1 or 2 below? Circle your answer.

1. I choose to do math because I like it.
2. If I had a choice I would not do math

D. Which statement do you most agree with, 1 or 2 below? Circle your answer.

1. I feel good when I do my math work.
2. I feel bad when I do my math work.

Teachley Survey - Administered during final research session for Study 1.

Name_____________ Date_________

Teachley

Make an X in **one** box that best describes what you think of **each statement**.

	1 **Not at All True**	**2**	**3** **Somewhat True**	**4**	**5** **Very True**
1. If I had a choice, I would choose to use the Teachley software during math.					
2. The Teachley software helped me learn math.					
3. When I run into difficulty doing a math problem, the Teachley software lets me go back and work out where I went wrong.					
4. The Teachley software was easy for me to use.					

Student Survey - Administered during the student focus group to assess students' feelings of interest, relevancy, and relatability (format adapted from TIMMS, 2019).

Student survey

How much do you agree with these statements about the math stories?

*Fill in only **one** circle for each row.*

	Agree a lot	Agree a litte	Disagree a little	Disagree a lot
The story topics (cooking, sports, animals) interest me	①	②	③	④
The stories would never happen in real life.	①	②	③	④
I know what the words in the stories mean.	①	②	③	④
The stories describe things I do now or will do as I get older.	①	②	③	④
I think the words in the stories are confusing.	①	②	③	④
I think my classmates would understand the stories.	①	②	③	④
The math stories describe things my friends, family, or neighbors do.	①	②	③	④
I understand what the stories are asking.	①	②	③	④
I want my teacher to use stories like these during math.	①	②	③	④

Appendix B - References

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Appendix C - About the Authors

Rachael Labrecque, PhD is Cofounder of Teachley and leads the company's school and district partnerships, with an emphasis on building strong, collaborative relationships to best support classroom implementation. She received a PhD in cognitive psychology from Columbia University where she studied elementary teacher preparedness to integrate technology into classroom instruction. With a background in early childhood education, she has extensive experience conducting observations of classroom instruction, developing teacher action plans, and co-leading professional development workshops. She is passionate about designing effective classroom materials that can be easily implemented in diverse classroom environments to support students ***and*** their teachers.

Kara Carpenter, PhD is Cofounder of Teachley and leads the product design and development. She was a National Board Certified Teacher with over 10 years of experience teaching children of all ages as a classroom teacher, outdoor education instructor and Peace Corps Volunteer. She has a PhD in Cognition and Learning from Teachers College, Columbia University, where her research focused on teaching mathematical strategies to young children using games. Her dissertation became the game *Addimal Adventure*, which won an Apple Design Award for being one of the 12 most innovative apps launched in 2014. She has been principal investigator on six of Teachley's Small Business Innovation Research grants from the Institute for Educational Sciences, National Science Foundation, and National Institutes of Health.

Dana Pagar, PhD is Cofounder of Teachley and has served as principal investigator on four of the company's SBIR grants. She has over 15 years of experience teaching and tutoring elementary school children from diverse backgrounds and is passionate about using technology to solve educational problems. She earned a PhD from Teachers College, Columbia University, where she studied the effects of digital manipulatives on children's mathematical development.

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